Abstract—This paper introduces a new mathematical model of positive leader propagation speed based on intense ionization in a thin boundary layer ahead of the tip of the leader channel. The model explicitly accounts for the effect of overvoltage and the resulting increase in leader current on leader speed. The model was applied to large rod-plane gaps under critical switching impulses. Numerical assessment of the model findings is included and whenever possible comparison with experiments has been found satisfactory.

Index Terms—Air gaps, leader, streamer, switching impulses.

I. INTRODUCTION

Investigation of the mechanism of positive leader propagation under slow front voltages is of great importance for basic understanding and practical applications in such phenomena as switching impulse breakdown of long air gaps [1]. The leader is basically an arc-like discharge with a voltage gradient that decreases with current and also with leader length or life time. Experiments by Les Renardières Group [2] on long air gaps under positive switching impulses of critical fronts showed positive leader currents in the range of 0.5–1.0 A and propagation speeds of 1–2 cm/μs.

Extensive investigations of the development of the positive spark in long air gaps, including the leader propagation phase, have been reported in [3]. Despite the merits of the model, presented as self-consistent, it still has some limitations: The work is based on detailed modeling by Gallimberti of an idealized single filament streamer. Furthermore, practical application involves a seemingly arbitrary choice of the number of filaments to obtain the real streamer charge. It is also noted that the number of filaments needed varies with experimental conditions. Although the model succeeds to account for several experimental facts, it was not as successful in predicting the effects of overvoltages on the final jump. Finally the model could only account qualitatively for the effect of overvoltages on positive leader speed for gaps even in the limited range of 4 m to 10 m.

A detailed investigation of thermal effects in air in the vicinity of the positive leader tip was reported in [4]. It was found that air heating in front of the leader tip could not account for the temperature range of 1500 K to 2000 K required in [3] for leader formation. Although the model described in [4] provided insight into some of the physical processes taking place near the tip of the leader, the leader speed was taken from measurements and could not be predicted from the model.

It is observed from the above that there is a real need for a model for positive leader speed suitable for engineering applications. The model should be capable of accounting for the effects of overvoltage and the resulting currents on positive leader speed. Although several simplifications will necessarily be introduced as explained below, the model must conform to known facts about the physics of the discharge phenomena involved.

II. MODEL FORMULATION

Leader propagation can only be sustained by appropriate current produced by streamer discharge (leader corona) which in turn is coupled to the electric field downstream from the leader tip. Criteria for positive leader formation and propagation have been formulated in [5]. Conditions at the leader tip are governed by the electric field due to the applied voltage which is opposed by the positive space charge field produced by leader corona. The effect of the space charge field is so substantial that it can completely shut down ionization phenomena in the leader tip zone (dead time).

In [5], the two criteria adopted for continuous leader formation and propagation are:

1) For stem formation, the streamers (leader corona) have to attain a critical size characterized by a streamer charge $Q_0$ related to the leader inception voltage by

$$Q_0 = c \cdot U_{lc}$$  \hspace{1cm} (1)

where $U_{lc}$ is the continuous leader inception voltage and $c$ is a constant independent of the gap length.

2) For continuous leader propagation, the resultant electric field $E_0$ at the leader tip has to reach a critical value $E_c$, i.e.,

$$E_0 = E_{app} - E_{sc} = E_c$$  \hspace{1cm} (2)

where $E_{app}$ is the field due to the applied voltage and $E_{sc}$ is the field due to the streamer space charge.

Since during continuous leader propagation resultant field conditions at the leader tip must be such that extinction of any streamer can be immediately replaced (second corona), we will in this model require that

$$E_c = E_{ci}$$  \hspace{1cm} (3)
where $E_{ci}$ is the corona inception field at the leader tip, which is determined by the initial radius $r_0$ of the leader core at continuous leader inception. Applying the streamer theory, the following relationship was obtained [6] between the corona inception field $E_{ci}$ and the equivalent electrode radius of curvature $r$

$$E_{ci}(r) = 2300 \cdot \left[ 1 + \frac{0.224}{r^{0.37}} \right] \text{ (kV/m, m)} \quad (4)$$

with $E_{ci}$ in kV/m and $r$ in m. In this model it is further assumed that leader extension during propagation is due to intense ionization in a thin boundary layer ahead of the leader tip, as shown schematically in Fig. 1. The injection into the leader tip of a charge $q$ per unit length will result in the increase of the conductance of the boundary layer from an initial value per unit length $g_0$ ahead of the leader tip to a value $g_c$ corresponding to a critical volume conductivity $\sigma_c$, characterizing the initial segment of the leader behind the leader tip. A quantity of interest is the thickness $b$ of the boundary layer. In analogy with heat transfer and fluid dynamics [7], the thickness of the ionization boundary layer will be defined by

$$b = \frac{\alpha_e}{\left| \frac{d \alpha_e}{dz} \right|_{z=0}} \quad (5)$$

where $z$ is the axial distance downstream from the leader tip, and

$$\alpha_e = \alpha - \eta \quad (6)$$

is the effective ionization coefficient at the leader tip. In this expression, $\alpha$ is the ionization coefficient and $\eta$ is the attachment coefficient, both being functions of the reduced electric field $E(z)/p$. Where $p$ is the ambient pressure.

For creation of a segment of the leader equal to the thickness of the ionization boundary layer $b$, the amount of the injection charge involved will be $bq$. The relationship between this amount of charge and the initial and final conductances per unit length $g_0$ and $g_c$, respectively, will for simplicity be expressed as

$$b \cdot q = k_0 (g_c - g_0). \quad (7)$$

In this expression, $k_0$ is Toepler’s constant, which was initially introduced as an empirical constant [8] but was more recently given a simplified physical interpretation in terms of the ionization process [9]. In the spark theory of Weizel and Rompe the conductance per unit length depends on the internal energy rather than the charge [10]. As shown in the Appendix, using Weizel and Rompe’s theory under a quasi-stationary current with $g_0 \ll g_c$, expression (7) will remain practically the same. We used the Toepler constant moreover because of the availability of reliable measurements [9]. In air at atmospheric pressure $k_0$ has been experimentally determined as $5 \times 10^{-3} \text{ Vs/m}$ [9].

For a newly formed leader segment, the voltage gradient starts at an initial value close to the streamer voltage gradient $E_{s\alpha}$ (4–5 kV/cm) so that the leader current can be expressed as

$$i = g_c E_{s\alpha} = \pi r_0^2 \sigma_c E_{s\alpha} \quad (8)$$

where $r_0$ is the initial radius of the leader and $\sigma_c$ is the critical volume conductivity.

Furthermore the leader current is related to the leader speed $v$ and the injection charge $q$ per unit length by

$$i = q \cdot v. \quad (9)$$

Substituting from (8) and (9) into (7), an expression of the leader speed is obtained

$$v = b E_{s\alpha} / [k_0 (1 - g_0/g_c)] \quad (10)$$

where $b$ is the thickness of the ionization boundary layer expressed by (5).

Under normal conditions, $g_0 \ll g_c$ so that (10) could be further simplified to

$$v = b E_{s\alpha}/k_0 \quad (11)$$

In the ionization boundary layer, in the immediate proximity of the leader tip, the reduced electric field will be initially high such that $E/p > 60 \text{ V/cm} \cdot \text{Torr}$, where attachment can be neglected. Based on measurements by Masch and Sanders, the following empirical expression [11] for the effective ionization coefficient can be used:

$$\alpha/p = 9.68 \exp(-264p/E) \quad (12)$$

With $\alpha$ in $\text{cm}^{-1}$, $p$ in Torr (133.32 Pa), and $E$ in V/cm.

For an electric field $E_{ci}$ at the tip of the hemispherically capped cylindrical leader of radius $r_0$ (see Fig. 1), it can be easily shown that the electric field gradient amounts to $-2E_{ci}/r_0$. Using (12), expression (5) can be expressed as

$$b = 2.49 \times 10^{-8} r_0 \cdot E_{ci} \text{ (m, V/m)} \quad (13)$$

with $b$ in m, $r_0$ in m and $E_{ci}$ in V/m.

An alternative approach that was explored for the determination of the thickness of the ionization boundary layer was to require that $b$ satisfies the condition

$$\alpha_e \cdot b = 18 \quad (5a)$$

with $\alpha_e$ in $\text{m}^{-1}$ and $b$ in m, which resembles the criterion for creation of a critical avalanche. In (5a), $\alpha_e$ corresponds to the resultant electric field at the leader tip, $E_{ci}(r)$.

For a 10-m leader-plane gap and leader radii of curvature $r$ in the range 0.8–2.4 mm (corresponding to overvoltages $U/U_{hi}$ in the range 1.0–3.0, as shown in the next section), Table I shows...
a comparison between the values of \( h \) obtained from (5) and (5a), confirming good agreement of both approaches. We have however maintained the application of (5) from before.

It should be noted that the leader channel diameter grows with time (due to energy dissipation) and therefore measurements on a long air gap, depending on the measuring position, will tend to be higher than for shorter gaps [12]. Since we are here interested in the leader core diameter in the immediate vicinity of the leader tip, leader channel diameter measurements for shorter gaps will be more appropriate. Of particular interest are measurements under critical front impulses. Measurements at the University of Munich in a 1.5-m rod-plane gap gave leader channel diameter in the range of 1–2 mm, [12] i.e., \( r_0 \) in the range of 0.5–1.0 mm. Similar information on initial leader diameter was also obtained by Schlieren technique [13].

### III. EFFECT OF OVERVOLTAGE AND LEADER CURRENT

Based upon experimental evidence in the laboratory, positive leader speed can be increased either by applying overvoltages to the air gap concerned [14] or by creating an electrically conducting filament along the path of the leader propagation [15]. The effect of the conducting laser filament on leader speed can in principle be accounted for by the increase of \( g_0 \) in (10) but this however is outside the scope of the present paper. This section will therefore only address the effect of overvoltages and the resulting current increase on the positive leader speed.

With a voltage \( U \) applied to the leader tip, the leader corona charge can be expressed as [16]

\[
Q(U) = A \cdot U(U - U_{ci})
\]

(14)

where \( A \) is in general a function of the distance from the leader tip to ground.

For leader corona, \( U \) is normally much larger than \( U_{ci} \) so that for any gap length the dependence of \( Q \) on \( U \) will approximately follow a quadratic law. Considering (1), \( Q \) can be expressed as

\[
Q(U) = \frac{eU^2}{U_{lc}} = Q_0 \left( \frac{U}{U_{lc}} \right)^2
\]

(15)

which means that \( Q \) will be proportional to the square of the overvoltage factor defined as \( U/U_{lc} \), with \( U_{lc} \), a function of the gap length [5].

We will further assume that the initial leader (stem) cross section will be proportional to the streamer charge \( Q \) (leader corona). In [13] there is experimental evidence that the leader cross section is approximately proportional to the total injected charge. From the previous assumption and (15) above it follows that for any overvoltage factor \( U/U_{lc} \):

\[
r(U) = r_0(U/U_{lc})
\]

(16)

It also follows that at leader inception, the initial leader cross section will be proportional to the critical charge so that with view of (1):

\[
r_0 = \text{const} \cdot \sqrt{U_{lc}}
\]

(17)

With a constant volume conductivity \( \sigma_v \), the initial leader conductance per unit length will be proportional to the initial leader cross section, so that

\[
i(U) = i_0(U/U_{lc})^2
\]

(18)

where \( i_0 \) is the continuous leader inception current, without overvoltage.

For any overvoltage factor \( U/U_{lc} \) and corresponding leader current \( i(U) \), the ionization boundary layer thickness, following (13), will be:

\[
b(U) = 2.49 \times 10^{-5} r(U) \cdot E_{ci}(r) \quad (\text{m}, \text{V/m})
\]

(19)

with \( b \) in m, \( r \) in m, \( E_{ci} \) in V/m, and with \( r(U) \) obtained from (16) with a known value of \( r_0 \).

At any overvoltage factor \( U/U_{lc} \), the leader speed will be obtained by replacing \( b \) in (11) by \( b(U) \):

\[
\nu(U) = b(U)E_0/k_0.
\]

(20)

It should be emphasized that in the above analysis, \( U \) refers to the voltage applied to the leader tip. The total applied voltage normally referred to in laboratory experiments includes moreover the leader voltage drop. Furthermore, \( U_{lc} \) here refers to leader inception voltage of the gap extending from the leader tip to ground. At leader inception, there is no difference with the conventional definition. However, as the leader progresses into the gap the distance \( h \) from the leader tip to ground is progressively reduced with a corresponding reduction of \( U_{lc}(h) \).

The present definitions were adopted for simplicity and more importantly for ease of application for very long gaps associated with lightning.

### IV. NUMERICAL EVALUATION AND COMPARISON WITH EXPERIMENTS

Since in initial leader channel diameter measurements by Les Renardières Group, [12] it was noted that any kind of distortion would produce an image broadening effect, we will adopt the minimum value of 0.5 mm for \( r_0 \) and a rod-plane gap of 1.5 m. Since according to [1] the leader inception voltage of a 1.5-m gap amounts to approximately 433 kV, the relationship between channel radius and the leader inception voltage for any gap length would be

\[
r_0 = 0.5 \sqrt{U_{lc}/433} \quad (\text{mm}, \text{kV})
\]

(21)

with \( r_0 \) in mm and \( U_{lc} \) in kV.
A sample model result is shown in Table II for a 10-m rod-plane gap both at continuous leader inception as well as under an overvoltage of 2 p.u. (with $E_{0} = 4 $ kV/cm).

As shown in [1], the continuous leader inception voltage of a rod-plane gap of length $d$ is given by

$$U_{i} = \frac{1.556}{(1 + 3.89/d)} \text{ (kV, m)} \quad (22)$$

with $U_{i}$ in kV and $d$ in m.

With $U_{i}$ of 1120 kV [1], $q_{0}$ of 45 $\mu$C/m [2], $r_{0} = 0.8$ mm, which results in $E_{c}$ of 95 kV/cm, boundary layer thickness $b$ of 190 $\mu$m a leader speed $v_{0}$ of 1.52 cm/$\mu$s, is obtained in excellent agreement with experiment [2].

With an overvoltage factor of 2 defined here by $U/U_{i}$, Table II shows an increased radius of the leader of 1.6 mm, a correspondingly reduced $E_{c}$ of 79 kV/cm, an increased ionization boundary layer thickness of 316 $\mu$m, an increased leader speed of 2.53 cm/$\mu$s, a substantially increased leader current of 2.74 A and a leader charge per unit length which reaches 108 $\mu$C/m.

It is generally accepted [1], [2] that for a rod-plane gap of length $d$ tested with a switching impulse with a critical front without overvoltage, the positive leader penetrates the gap with practically constant tip potential equal to the continuous leader inception voltage $U_{i}(d)$. As mentioned before, however, as the leader approaches the plane, the distance $h$ from the leader tip to plane decreases continuously with corresponding decrease of the voltage $U_{i}(h)$ needed for leader inception of such a gap. Therefore the ratio between the tip potential $U_{i}(d)$ and $U_{i}(h)$ will be greater than unity, providing a virtual overvoltage according to the definition of the present model. The model therefore predicts that even without application of an overvoltage to the high voltage electrode, the leader speed increases slightly as the leader progresses towards ground. This is shown in Fig. 2, for a 10-m rod-plane gap exposed to a critical positive switching impulse, for axial leader length up to 6.5 m. In the same range of the leader length the current increased in a range of 0.69--1.36 A.

For overvoltage factors in the range 1.0--2.4 p.u., Fig. 3 shows the variation of the leader current $i$ for a 10-m rod-plane gap. The leader current $i$ varies from approximately 0.7 A to 4.0 A. In the same overvoltage range the positive leader speed varies from 1.52 cm/$\mu$s to 2.89 cm/$\mu$s, as shown in Fig. 4, confirming that the leader current is much more sensitive to overvoltages than leader speeds [14]. Regression analysis (R square: 0.99999) in the same overvoltage range resulted in the following relationship between leader speed and leader current for the 10-m rod-plane gap under consideration:

$$v = 1.75 \times 10^{4} \cdot i^{0.367} \text{ (m/s, A)} \quad (23)$$

with $v$ in m/s and $i$ in A. Extending the overvoltage factor, for the 10-m gap, to cover the range 1.0--3.0 p.u., expression (23) remains practically unchanged. Expression (23) also proved to be rather insensitive to the gap length. For a 4-m rod-plane gap and overvoltages in the range 1.0--2.0 the exponent amounted to 0.364 and the speed corresponding to a leader current of 1 A amounted to 1.61 $\times$ 10$^{4}$ m/s.

The following empirical relationship by Hutzler [17] was based on measurements by Les Renardières Group:

$$v = 1.75 \times 10^{4} \cdot i^{0.30} \text{ (m/s, A)} \quad (24)$$

with $v$ in m/s and $i$ in A. Comparison between model results and the above empirical formula are shown in Fig. 5 and considered very satisfactory.

Regression analysis of model results (R square: 0.99999) for the 10-m gap also shows that dependence of the positive leader speed on the overvoltage factor defined before takes the form

$$v = 1.52 \times 10^{4}(U/U_{i})^{0.736} \text{ (m/s)} \quad (25)$$

with $v$ in m/s and $U$ and $U_{i}$ in kilovolts. Finally, Fig. 6 shows the effect of gap length $d$ on leader speed at inception for gaps
in the range 5–30 m. It is shown, in agreement with experiment, [2] that the leader speed at inception varies in a limited range despite the wide range of gap lengths investigated. Note that for laboratory experiments on gaps of practical dimensions, the overvoltage factor, normally defined as the ratio between the test voltage and the 50% breakdown voltage [14] can only be varied within a limited range above which streamer breakdown of the gap takes place.

V. CONCLUSION

A new model for the speed of a positive leader discharge has been introduced and applied to long air gaps under critical switching overvoltages.

1) The model is based on intense ionization in a thin boundary layer ahead of the leader tip.

2) In order to replace any possible extinction of a streamer filament, the net electric field at the leader tip due to both the applied voltage and the charge of leader corona has to be maintained at or above the corona inception field.

3) The thickness of the ionization boundary layer has been defined by the ratio between the net ionization coefficient $\alpha_c$ and its space gradient $\partial \alpha_c/\partial z$ at the leader tip.

4) The transition from an initial conductance $g_0$ per unit length ahead of the leader tip to the critical conductance

\[ \frac{\dot{g}^2}{\pi r^2 \sigma} = \frac{d}{dt}(\pi r^2 u) \]  

in the range 5–30 m. It is shown, in agreement with experiment, [2] that the leader speed at inception varies in a limited range despite the wide range of gap lengths investigated. Note that for laboratory experiments on gaps of practical dimensions, the overvoltage factor, normally defined as the ratio between the test voltage and the 50% breakdown voltage [14] can only be varied within a limited range above which streamer breakdown of the gap takes place.

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4) The transition from an initial conductance $g_0$ per unit length ahead of the leader tip to the critical conductance
where \( i \) is the current at any instant, \( r \) is the channel radius, \( \sigma \) is the electrical conductivity of the channel, \( u \) is the internal energy per unit volume.

Simplified analysis by Weizel and Rompe led to the following relationship between \( \sigma \) and \( q \):

\[
\sigma = au/p \tag{27}
\]

where \( p \) is the ambient pressure and \( q \) is the Weizel and Rompe’s spark constant, which is independent of the pressure.

Considering the conductance per unit length

\[
g = \pi r^2 \sigma \tag{28}
\]

and substituting in (26) from (27) and (28), we obtain

\[
\frac{\dot{i}^2}{g} = \frac{p}{\pi r^2} \frac{dg}{dt} \tag{29}
\]

Integrating (29) for a constant current from the initial value \( g_0 \) to the critical value \( g_c \)

\[
\dot{i}^2 \Delta t = \frac{p}{2\pi} (g_c^2 - g_0^2) \tag{30}
\]

where \( \Delta t \) is the time step for the leader to span the ionization boundary layer of thickness \( b \).

Since the injected charge per unit length is \( q \), it follows that:

\[
q \cdot b = i \cdot \Delta t. \tag{31}
\]

Recognizing that \( q \Delta t \ll g_c \), it follows from (30) and (31) that:

\[
i \cdot q \cdot b = \frac{p}{2\pi} g_c (g_c - g_0). \tag{32}
\]

Multiplying both sides of (32) by the initial leader gradient \( E_s \) and recognizing from (8) that

\[
i = g_c E_s \tag{33}
\]

(32) becomes

\[
q \cdot b = \frac{p}{2\pi E_s} (g_c - g_0) \tag{34}
\]

which is identical to (7) with Toeppler’s constant \( k_0 \) being replaced by \( p/(2\pi E_s) \).

REFERENCES


Farouk A. M. Rizk (SM’75–F’82–LF’05) was born in Gharbia, Egypt, on July 6, 1934. He received the B.Sc.Eng. and M.Sc. degrees from Cairo University, Cairo, Egypt, in 1955 and 1958, respectively, the L.Tech. degree from the Royal Institute of Technology, Stockholm, Sweden, in 1960, and the D.Tech. degree from Chalmers University of Technology, Gothenburg, Sweden, in 1963.

He was a Research Engineer with ASEA (now ABB), Ludvika, Sweden, in the High Power Laboratory, Ludvika, Sweden, from 1960 to 1963, and the Computer Department, Vasteras, Sweden in 1963. He was with the Egyptian Electricity Authority from 1964 to 1971, becoming Manager of High Voltage in 1968. He joined the Institut de Recherche d’Hydro-Québec (IREQ) as a Senior Research Scientist in 1972, becoming Program Manager in 1975, Scientific Director in 1976, Director of Power Transmission in 1980, andInterruptor President in 1986. He was Fellow Research Scientist at IREQ from 1987 to 1996. Since 1997, he has been President of the consulting firm Expodex, Inc.

Dr. Rizk was awarded the Egyptian National Prize of Engineering Science for 1971 and decorated by the Order of Worthiness (Third Class) in 1972 and the Order of Science (First Class) in 1973. He is a Registered Professional Engineer in the Province of Quebec and has been International Chairman of Technical Committee 28: Insulation Coordination, of the International Electrotechnical Commission (IEC) from 1984 to 1996 and Convener of CIGRE Working Group on Insulator Pollution from 1988 to 2002.

François Vidal received the Ph.D. degree in subatomic physics from the University of Montréal, Montréal, QC, Canada, in 1988.

In 1991, he was a Computational Physicist with the Institut National de la Recherche Scientifique-Université du Québec, Varennes, QC, Canada, where he became a Professor in 2003. His current fields of research are mainly related to laser-matter interaction, including laser-triggered electrical discharges, laser-induced plasma spectroscopy, propagation of intense ultrashort laser pulses in optical media, and the generation of attosecond radiation from solid targets.